A 3D ESTIMATION OF AIRSIDE CAPACITY AT AIRPORTS

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RESUMO
Nesta comunicação, o conceito de capacidade tridimensional para o "lado ar" de um aeroporto é introduzido. Uma nova abordagem, baseada na resolução de uma sequência de problemas de otimização, é proposta para a estimação da capacidade prática do "lado ar" de um aeroporto. A area de movimentação e estacionamento das aeronaves é modelizada por uma rede capacitada e os atrasos medios gerados pelas interações entre fluxos ao nivel das intersecções das vias de circulação são levados em consideração. Um primeiro problema de otimização é formulado para obter uma estimativa da capacidade téorica do "lado ar" de um aeroporto. Introduzindo então restrições de relacionadas com a operação efetiva do sistema e dados os niveis medios de chegadas e partidas para uma situação de ocupação das areas de estacionamento, um problema de otimização (de tipo minimização do custo total de um fluxo em rede) é formulado para obter uma estimativa da capacidade prática do "lado ar" de um aeroporto. Este problema de otimização sendo não convexo, devido à natureza do critério de otimização, um método de resolução de tipo heurístico é proposto: Este método carga progressivamente e de forma estocástica a rede de circulação e para cada nível, uma versão linearizada do problema é tratada. Este processo é repetido varias vezes, escolhendo no fim a melhor solução obtida. Este processo heurístico tem sido validado por simulação, apresentando resultados satisfactorios. Finalmente, resultados numêricos relativos ao aeroporto internacional de Portland são apresentados.

ABSTRACT
In this communication, a 3D concept of airside capacity is introduced. A capacitated network is built to represent the airside of an airport. Then a new approach based on the solution of successive optimization problems is proposed for the estimation of the practical airside capacity of an airport. First theoretical airside capacity can be estimated by solving a standard linear program in a network. Then, for given mean in-bound and out-bound flows and a current ground traffic situation, a constrained minimal cost flow problem is formulated. The interaction of aircraft flows at ground intersections (taxiways and aprons) is taken explicitly into consideration and this leads to a non convex optimization problem. A heuristic approach, based on a progressive loading of the network, is developed to get a solution for the minimal cost flow problem. The proposed heuristic is assessed numerically through extensive testing, showing acceptable performances. The proposed approach has been applied to the case of Portland International airport for which some numerical results are displayed.

1. INTRODUCTION
With the sustained development of air transportation over the last decades, airport capacity has remained a permanent issue for airport planners and operators. Until recently, airport capacity was considered only at its two traditional bottlenecks: the runways system capacity and the passenger's terminals capacity. However, today, aircraft ground traffic at airports has become also a critical question with important influences on security and efficiency levels and new ground traffic management and control systems including a higher degree of automation have been introduced. Traditionally, with respect to airside airport capacity , a distinction has been done between theoretical and practical capacity, depending if level of service thresholds and operational practices are taken into account or not. In general, practical capacity, which is

of main interest for airport managers, has been estimated on statistical grounds while cumbersome simulation models have been developed to perform some scenario based capacity predictions. It appears that since in-bound and out-bound flights are competing to use the same airport facilities the nature of this problem is multi criteria and capacity must be defined in terms of Pareto frontiers. Here different 2D capacity concepts are recalled (STBA, 1975):

- Theoretical Capacity: the maximum number of aircraft that the airport is able to process per unit of time without considering the quality of service..
- Potential Capacity: the maximum number of aircraft that the airport is able to process per unit of time for given levels of demand (arrivals).
- Practical Capacity: the maximum number of aircraft which can be processed per unit of time for a given mean delay level.
- Operational capacity: the maximum number of aircraft which can be processed per unit of time for a given maximum delay.

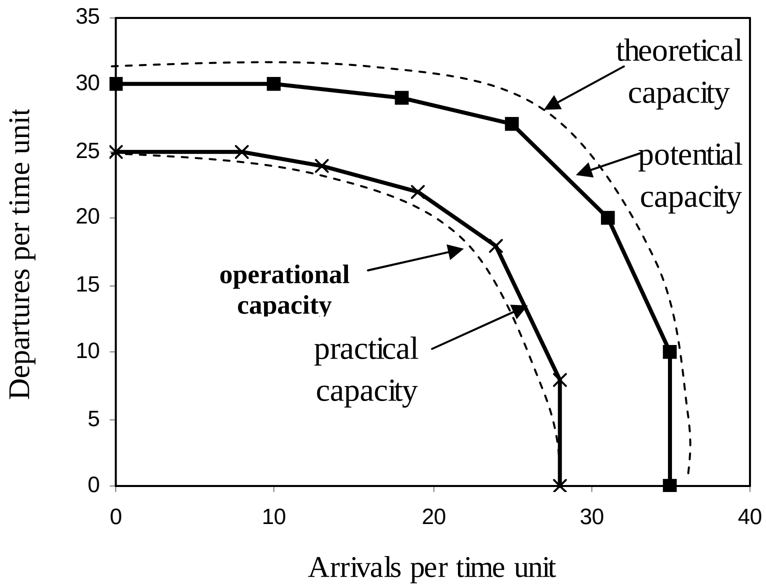

Figure 1: Different 2D airside capacities at airports

In this communication, a new dimension is introduced in the airside capacity analysis: the amount of grounded aircraft present in the airside active areas. This parameter can be significant with respect to airside capacity at major airports such as Heathrow, O'Hare or Schiphol, but also at some domestic airports such as Congonhas, Hannover or Orly.
An approach based on the solution of successive optimization problems is proposed to perform the 3D estimation of the practical airside capacity of an airport. For given mean in-bound and out-bound flows and a current ground traffic situation, a minimal cost flow problem is formulated. The interaction of aircraft flows at ground intersections (taxiways and aprons) is taken explicitly into consideration and this leads to a non convex optimization problem. A heuristic approach, based on a progressive loading of the network, is developed to get a solution for the minimal cost flow problem. The proposed heuristic is assessed numerically, showing acceptable performances. Out-bound flows are increased in the

optimization problem until practical capacity levels are reached, at a point of the practical capacity Pareto frontier.

The proposed approach has been applied to different case studies. In this communication, some numerical results relative to its application to the case of Portland International Airport are displayed.

2. AIRSIDE TRAFFIC NETWORK MODELLING

The airside at airports is composed of three main components: the traffic network, composed of runways, taxiways, aprons and parking areas, the flows of aircraft and the ground traffic signalling and control system. The main objective is to allow ground traffic operations at minimum costs, avoiding saturation problems while insuring high standards of security. Many management and control issues lead to decision problems whose solutions can be based on suitable models of grounds traffic operations. Various models (Andreatta, 1998) have been built with this purpose. They are different with respect to the time period considered (long term models are devoted to the design of the system, medium term models are involved with airport operations planning and short term models are related with activity control issues), to the level of detail (macroscopic, intermediate or microscopic for each of the main components of the traffic system) and to the degree of determinism adopted. However, none of them allows the estimation of airside capacity at airports taking into account the aircraft ground traffic conditions.

2.1 Airside network modelling

Here a reference period of time of about an hour is retained so that arrival and departure rates can be considered to be steady. The waiting queues present at different stages of the traffic system are not considered explicitly but the storage capacities in different sections of this traffic system are taken into account. Aircraft ground traffic is considered to flow continuously from runways to parking positions and from parking positions to runways, while the stock of parked aircraft is also considered. Here an oriented graph $G=(N,U)$, is used to represented in a rather accurate way the airport aircraft ground circulation network:

- The set of nodes N of the graph represent connection points and the limits of the circulation ways and can be classified according to their functions : mere connecting points between two successive segments, crossing point with competing traffic, decision points.

Special nodes (runway entries, runway exits and crossing points) must be also introduced to represent runways.

- The set of arcs U of the graph is composed of five sub-sets : runways, runways exit segments, taxiways, aprons and parking areas. Some of these arcs can be bidirectional, while the orientation of some others are dependent of the current mean wind direction. This arc orientation can obey to either a group logic (runways arcs) or a local one (bidirectional arcs and parking positions). To each arc different parameters are attached : length and width, maximum wingspan, maximum weight, storage capacity, orientation, etc.

- The geometry of the parking positions, of the taxiways crossings and of the aprons leads to additional constraints forbidding the use of determined types of aircraft in some areas of the traffic system. Thus to each aircraft type a circulation sub graph can be defined. We have :

$$U = U_{ad} \cup U_a \cup U_d \tag{1}$$

where U_a is the set or arcs involved related with arriving traffic, U_d is the set of arcs related with departing traffic, U_{ad} is the set of arcs associated with circulation segments used both for departures as for arrivals. U' is the set of double oriented arcs : $U' = \{u \in U \mid \forall\, u = (a,b),\, \exists u' = (b,a), a,b \in N\}$, I is the set of runways under operation during the time period considered, R_i^s , $i \in I$, is the set of arcs associated to the runway exits, R_i^e , $i \in I$, is the set of arcs associated to the run ways entries, J is the set of available parking areas, P_j^s , $j \in J$, is the set of arcs associated to the exits from the parking areas, P_j^e $j \in J$, is the set of arcs associated to the entries of the parking areas.

2.2 Traffic flow modelling

The traffic flow through an arc is defined here as the number of aircraft movements per period of time through the corresponding circulation link. Arriving and departing flows are considered separately : Φ^a is the total arriving flow from the runways and Φ^d is the total departing flow towards the runways, φ_u^a and φ_u^d , are the arriving and departing flows using arc u. To each arc u is attached a capacity whose level φ_u^{max} is related with its geometric characteristics and to the size and operational characteristics (taxiing speed). This leads to a set of arc capacity and positive ness constraints:

$$0 \le \varphi_u^a + \varphi_u^d \le \varphi_u^{max}, u \in U_{ad} , 0 \le \varphi_u^a \le \varphi_u^{max}, u \in U_a , 0 \le \varphi_u^d \le \varphi_u^{max}, u \in U_d , \tag{2}$$

$$0 \le \varphi_u^a \le \varphi_u^{max}, u \in R_i^s, i \in I , \qquad 0 \le \varphi_u^d \le \varphi_u^{max}, u \in R_i^d, i \in I \tag{3}$$

$$0 \le \varphi_u^a \le \varphi_u^{max}, u \in P_j^e, j \in J , \qquad 0 \le \varphi_u^d \le \varphi_u^{max}, u \in P_j^s, j \in J \tag{4}$$

where φ_u^{max} is the maximum flow for arc u.

The network definition is completed by the addition of flow conservation constraints at the different crossings and decision points. So we introduce the following constraints :

$$\sum_{u \in \omega^-(i)} (\varphi_u^a + \varphi_u^d) + \sum_{v \in \omega^-(i)} \varphi_v^d + \sum_{w \in \omega^-(i)} \varphi_w^a = \sum_{u \in \omega^+(i)} (\varphi_u^a + \varphi_u^d) + \sum_{v \in \omega^+(i)} \varphi_v^d + \sum_{w \in \omega^+(i)} \varphi_w^a \tag{5}$$

where $u \in U, v \in U_d, w \in U_a$, $\omega^-(i)$ is the set of incident arcs to node i, $\omega^+(i)$ is the set of leaving arcs from node i. Other flow conservation constraints are attached to the exits and entries of the runways and parking areas :

$$\sum_{u \in R_i^s} \varphi_u^a = \Phi_i^a, \forall i \in I \text{, where } \Phi_i^a \text{ is the landing flow at runway } i \tag{6}$$

$$\sum_{u \in R_i^e} \varphi_u^d = \Phi_i^d, \forall i \in I \text{, where } \Phi_i^d \text{ is the take-off flow from runway } i \tag{7}$$

$$\sum_{u \in P_j^s} \varphi_u^d - \Psi_j^d = 0, \forall j \in J \text{, where } \Psi_j^d \text{ is the departing flow from parking area } j \tag{8}$$

$$\sum_{u \in P_j^e} \varphi_u^a - \Psi_j^a = 0, \forall j \in J \text{, where } \Psi_j^a \text{ is the arriving flow at parking area number } j. \tag{9}$$

The transfer from one link to another happens at intersections where other concurrent flows may be present, this is taken into account through the following constraints :

$$\sum_{u \in \omega^-(l)} \left(\varphi_u^a + \varphi_u^d\right)\theta_l \le T, \quad l \in L \tag{10}$$

where L is the set of such intersections, θ_l is the mean crossing time of intersection l. In the case of double oriented arcs, the capacity constraints are written as :

$$\sum_{u \in U'} \left(\varphi_u^a + \varphi_u^d\right)\tau_u + \sum_{u' \in U'} \left(\varphi_{u'}^a + \varphi_{u'}^d\right)\tau_{u'} \le T \tag{11}$$

where τ_u is the arc occupancy time in one direction and $\tau_{u'}$ is the arc occupancy time in the other direction. Other flow conservation constraints are respectively for parking areas exits, for parking areas entries, for entries to runways and for runways exits:

$$\sum_{j\in J}\Psi_j^d = \Phi^d\,,\qquad \sum_{j\in J}\Psi_j^a = \Phi^a\,,\qquad \sum_{i\in I}\Phi_i^d = \Phi^d\,,\qquad \sum_{i\in I}\Phi_i^a = \Phi^a \tag{12}$$

If the global flows Φ^a and Φ^d are input parameters for the capacity study, it is the model which should distribute them between the different parking areas and runways. Let I_0 be the set of runways whose landing and departing activities are independent from each other. The corresponding runways capacity constraints can be written as :

$$\lambda_\alpha^i \Phi_i^a + \gamma_\alpha^i \Phi_i^d \le C_i \;,\; \alpha \in A_i, i \in I_0 \tag{13}$$

where $\lambda_\alpha^i, \gamma_\alpha^i$ et C_j are real positive parameters, A_i is the set of indexes for the constraints defining a convex capacity domain for runway i. For the set I_k of interdependent runways, the convex capacity domain equations can be written as :

$$\sum_{i\in I_k}\left(\lambda_\alpha^{il}\Phi_i^a + \gamma_\alpha^{il}\Phi_i^d\right) \le C_l \;, \alpha \in A_{kl}, l \in L_k, k \in K \tag{14}$$

where A_{kl} and L_k are the set of indexes related with the k^{th} sub set of the K interdependent runways. The same modelling approach can be adopted in relation to the parking areas, however, here it will be assumed that the parking areas are independent. Then the parking capacity constraints can be written as :

$$0 \le N_j^0 + \Psi_j^a - \Psi_j^d \le S_j,\; j \in J \tag{15}$$

and

$$\sum_{j\in J} N_j^0 = N^0, \quad N_j^0 \ge 0, \forall j \in J \tag{16}$$

where S_j is the capacity of the j^{th} parking area, N_j^0 is the number of occupied positions in the j^{th} parking area at the beginning of the period, N^0 is the total number of occupied parking positions at the beginning of the period.

3. THEORETICAL CAPACITY EVALUATION

The theoretical capacity is given by the (Φ^a , Φ^d) Pareto frontier with $\underline{N}^0$ as parameter. It can be obtained by solving repeatedly problem $P(\Phi^a, \underline{N}^0)$ given by :

$$\max_{\underline{\varphi},\underline{\Phi},\underline{\Psi},\underline{N}} \Phi^d \tag{17}$$

$$\text{with}\quad \underline{\varphi} = \begin{pmatrix}\underline{\varphi}^a\\ \underline{\varphi}^d\end{pmatrix}, \underline{\Phi} = \begin{pmatrix}\underline{\Phi}^a\\ \underline{\Phi}^d\end{pmatrix}, \underline{\Psi} = \begin{pmatrix}\underline{\Psi}^a\\ \underline{\Psi}^d\end{pmatrix}, \underline{N} = \begin{pmatrix}\underline{N}_1^0\\ \underline{N}_J^0\end{pmatrix} \tag{18}$$

with (14), (15), (16) and the additional constraints :

$$\sum_{j\in J}\Psi_j^d = \Phi^d\,,\; \sum_{j\in J}\Psi_j^a = \Phi^a\,,\; \sum_{i\in I}\Phi_i^d = \Phi^d\,,\; \sum_{i\in I}\Phi_i^a = \Phi^a \tag{19}$$

$$\underline{\varphi} \in F(\underline{\Phi}, \underline{\Psi}, \underline{N}) \tag{20}$$

where $F(\underline{\Phi}, \underline{\Psi}, \underline{N})$ is the convex set defined by the traffic flow capacity constraints expressed with $\varphi_u^a, \varphi_u^d, u \in U$ for given levels of $\underline{\Phi}, \underline{\Psi}$ et $\underline{N}$. Here the flow variables are taken real so that large scale integer linear programming problems are avoided. Let us notice that the set of flow related constraints $\underline{\varphi} \in F(\underline{\Phi}, \underline{\Psi}, \underline{N})$, is decoupled from the runway and parking area capacity constraints of problem $P(\Phi^a, \underline{N}^0)$. The connection between the two sets of variables is realized by the global flows $\underline{\Phi}, \underline{\Psi}$ and $\underline{N}$. If in a first step, the flow related constraints which are a capacity limiting factor, are let apart, the following relaxed problem can be formulated. $\tilde{P}(\Phi^a, \underline{N}^0)$:

$$\max_{\underline{\Phi},\underline{\Psi},\underline{N}} \Phi^d \tag{21}$$

under constraints (14),(15),(16),(19) and with

$$\Psi_j^a \geq 0, \Psi_j^d \geq 0,\ j \in J\ ,\quad \Phi_i^a \geq 0, \Phi_i^d \geq 0, i \in I \tag{22}$$

The solution of this problem, $\tilde{\Phi}^d$, will be an upper bound for the solution of problem $P(\Phi^a, \underline{N}^0)$. If there is a feasible flow $\underline{\varphi}$ for $F(\underline{\tilde{\Phi}}, \underline{\tilde{\Psi}}, \underline{\tilde{N}})$ where $(\underline{\tilde{\Phi}}, \underline{\tilde{\Psi}}, \underline{\tilde{N}})$ is the solution of $\tilde{P}(\Phi^a, \underline{N}^0)$, then the Φ^d solution of problem $P(\Phi^a, \underline{N}^0)$ is such that : $\Phi^{d^*} = \tilde{\Phi}^d$. In this case, (N^0, Φ^a), the circulation system is not a limiting factor for the airside capacity which in that case is only dependant of the capacities of the runways systems and parking areas. In the case where $F(\underline{\tilde{\Phi}}, \underline{\tilde{\Psi}}, \underline{\tilde{N}})$ is an empty set, the circulation flow constraints must be taken into account to evaluate the airside capacity. In this case it is necessary to cope with the global linear programming problem $P_\varphi(\Phi^a, \underline{N}^0)$ which is equivalent to problem $P(\Phi^a, \underline{N}^0)$, but where the circulation flows appear explicitly in the global constraints related with runways and parking areas capacities. $P_\varphi(\Phi^a, \underline{N}^0)$ is then written :

$$\max_{\underline{\varphi},\underline{N}} \Phi^d \tag{23}$$

with the arc capacity and positive ness constraints (2), (3) and (4), the conservation constraints (5), the parking capacity constraints (10) and (11) and the runway capacity constraints (14) and with:

$$\sum_{j\in J} \sum_{u \in P_j^s} \varphi_u^d = \Phi^d \quad \sum_{j\in J} \sum_{u \in P_j^e} \varphi_u^a = \Phi^a \quad \sum_{i\in I} \sum_{u \in R_i^e} \varphi_u^d = \Phi^d \quad \sum_{i\in I} \sum_{u \in R_i^s} \varphi_u^a = \Phi^a \tag{24}$$

The above problem can be solved easily with a variation of the well known « out of kilter » algorithm. Then a possible solution scheme for the estimation of airside theoretical capacity is to solve repeatedly problem $P(\Phi^a, \underline{N}^0)$ for $N^0 \in \{0,..,N^0_{\max}\}$ and $\Phi^0 \in \{0,\ldots, \Phi^0_{\max}\}$ where N^0_{max} is the maximum number of parking positions that can be used simultaneously in the whole airport. Φ^a_{max} is the solution of problem $P(\Phi^a)$ given by :

$$\max \Phi^a \tag{25}$$

with
$$\sum_{i \in I_k} \lambda_\alpha^{il} \Phi_i^a \leq C_l, \quad \alpha \in A_{kl}, l \in L_k, k \in K \tag{26}$$

and
$$\sum_{i\in I} \Phi_i^a = \Phi^a \quad \text{with} \quad \Phi_i^a \geq 0, i \in I \tag{27}$$

The output of this heavy process is the theoretical airside capacity which can be represented in a three dimensions space (Φ^a, Φ^d, N^0) :

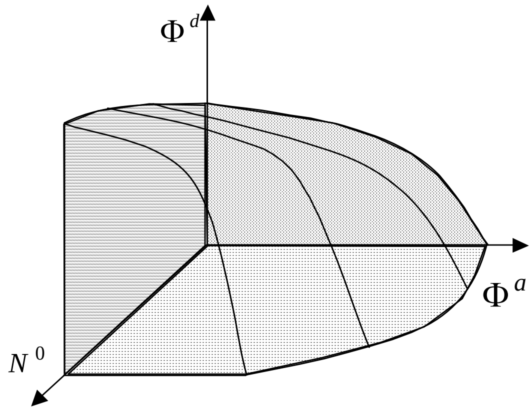

Figure 2: 3D theoretical airside capacity

The evaluation of the theoretical capacity does not take into account all the interactions between arriving and departing flows and the capacity of the ground traffic control sectors (The airside is divided in control sectors, each of them being operated by a ground controller). Then, practical airside capacity is, by far, smaller than theoretical capacity.

4. PRACTICAL CAPACITY EVALUATION

An operational situation, given by the arriving flows and the initial airside occupancy, $\Phi_i^a, i \in I$ et $N_j^0, j \in J$, is considered. Practical capacity is considered to be reached when for a global departing flow Φ^d, the mean ground delay is greater than a given threshold (15 minutes in general). Then, delays must be estimated to achieve the evaluation of airside practical capacity.

4.1 Traffic delay modelling

Considering that the mean travel time for an aircraft along a given arc is an increasing function of the flow through this arc, a model such as:

$$t_p(\varphi) = t_0\left(1 + c\cdot\left(\phi/\phi^m\right)^\alpha\right) \text{ with } \alpha > 1 \text{ and } c > 0 \tag{28}$$

could be adopted. When the flow reaches the capacity of the arc, a queue has build up over the whole arc and traffic is stopped. When the flow of the arc interacts with other flows, it is necessary to take into account the resulting delay. A mean delay model which can be adopted, since is complexity is medium while taking into account the main phenomena, is the following :

$$t_u(\varphi) = \frac{l_u}{\tilde{v}_u}(1+\alpha_u\varphi_u)^{\gamma_u} + \sum_{j\in\omega^-(u)}\varphi_j\tau_j \tag{29}$$

where l_u is the length of arc u, $\tilde{v}_u$ is the standard aircraft ground free speed for this class of arc, α_u and γ_u are parameters characteristic of proper congestion effects, $\omega^-(u)$ is the set of incident arcs towards u, τ_j is the mean additional delay resulting from side flow φ_j (this parameter should allow to take into account wait time at crossings, minimum separation standards and the relative orientation of arcs at crossings). In a more generic way, the mean travel delay along an arc u could be given by :

$$f_u(\varphi_u, \mathrm{F}_u^-) \text{ where } \mathrm{F}_u^- = \{\varphi_v \mid v \in \omega^-(i), v \neq u\} \tag{30}$$

4.2 Formulation of the airside traffic flow optimisation problem

Here, it is considered that the total arriving flow Φ^a is already given and that the initial distribution of grounded aircraft over the different parking areas is known. The problem is here to maximize the total departing flight while insuring that the travel delays remain under a given threshold. This problem, $P_{\Phi^d}(\Phi^a, \underline{N}^0)$, is written as :

$$\max_{\Phi^a, N_j^0} \Phi^d \qquad \text{with} \qquad \sum_{u\in U} f_u(\varphi_u, \mathsf{F}_u^-) / \sum_{u\in U} \varphi_u \le d_{max} \tag{31}$$

and with the flow constraints within the ground traffic network written in a summarized way as :

$$\underline{0} \le \underline{\varphi} \le \underline{\varphi}^{max} \qquad \text{and} \qquad \mathbf{A}\underline{\varphi} = 0 \tag{32}$$

where A is the arc-node incident matrix and d_{max} is an upper limit for the mean delay. Since this problem, due to the delay constraint, is not a standard max flow problem over a network, progressively increasing Φ^d until problem $P_{\Phi^d}(\Phi^a, \underline{N}^0)$ has no more a feasible solution. A bound with respect to mean delay is obtained from the solution of problem $P_f(\Phi^a, \underline{N}^0, \Phi^d)$, given by :

$$\min_u \sum_{u\in U} f_u(\varphi_u, \mathsf{F}_u^-) \tag{33}$$

with constraints (2),(3),(4),(5),(6),(7),(8),(9),(10),(11),15),(16) and with:

$$0 \le \Psi_j^a \le \Psi_j^{a\,max},\ j\in J \qquad \sum_{j\in J} \Psi_j^a = \Phi^a \tag{34}$$

$$0 \le \Psi_j^d \le \Psi_j^{d\,max},\ j\in J \qquad \sum_{j\in J} \Psi_j^d = \Phi^d \tag{35}$$

and

$$\sum_{u\in\Omega_k^{ad}} \left(\varphi_u^a + \varphi_u^a\right) + \sum_{v\in\Omega_k^d} \varphi_v^d + \sum_{w\in\Omega_k^a} \varphi_w^a \le Z_k, \quad k\in\{1,2,\ldots,K\} \tag{36}$$

where $\Psi_j^{a\,max}$ and $\Psi_j^{d\,max}$ are the maximum arriving and departing flows at parking area number j , $\Omega_k = \Omega_k^{ad} \cup \Omega_k^d \cup \Omega_k^a$ is the sub set of traffic links controlled by the k[th] ground traffic controller, the capacity of the corresponding control is Z_k. K is the total number of ground control sectors in the airport.

4.3. A solution approach to estimate practical airside capacity

Taking into account the extreme difficulty to cope with ground traffic delays (modelling and optimisation issues), a different approach can be proposed : Traffic scenarios can be generated by a simplified optimization process and then performance, considering traffic delays, can be evaluated more accurately through simulation studies. Following this approach, the optimization criterion can tackle in a simpler way the interaction between competing flows at crossings. An acceptable interaction index can be such as :

$$f(\underline{\varphi}) = \sum_{u\in U} l_u\varphi_u + \sum_{l\in L}\sum_{u\in\omega^-(l)}\sum_{\substack{v\in\omega^-(l)\\ v\ne u}} c_{uv}\varphi_u\varphi_v \tag{37}$$

where c_{uv} and d_u are positive parameters. Then the optimization problem to be solved in this case $P_{op}\left(\Phi^a, \underline{N}^0, \Phi^d\right)$, is given by :

$$\min \left(\sum_{u\in U} l_u\varphi_u + \sum_{l\in L}\sum_{u\in\omega^-(l)}\sum_{\substack{v\in\omega^-(l)\\ v\ne u}} c_{uv}\varphi_u\varphi_v\right) \tag{38}$$

under constraints (2),(3),(4),(5),(6),(7),(8),(9),(10),(11),(12),(15), (16),(34),(35) and (36).

The optimization criterion of this problem is not convex in general and the use of classical quadratic mathematical programming techniques could lead to local minima very different from the true solution. Also, global optimization techniques, which need in general large computing time to provide a candidate solution, could be applied, again without any assurance of optimality. Another way should be to modify the optimization criterion by introducing quadratic terms so that it becomes convex, however in this case, the representative ness of the criterion is largely lost. Here a new approach has been used: the solution is obtained through an iterative process in which the network is progressively loaded through a stochastic process, the criterion is linearized at each step of the loading, the resulting linear optimization problem is solved and the loading is pursued until completion of the arriving and departure traffics. A sequence of arriving and departing flows :

Φ^{a_i} et Φ^{d_i} , $i = 0,\ldots, Q$, such as : $\Phi^{a_i} = \lambda_i \Phi^a$ et $\Phi^{d_i} = \lambda_i \Phi^d$ (39)

with: $\lambda_i = 1 - \left(\left(Q - i - n\right)/\left(Q + i + n_i\right)\right)^p, \ i = 0,\ldots,Q$ (40)

where p is an integer and n_i is the result of an uniform random choice over the interval]0, 1].

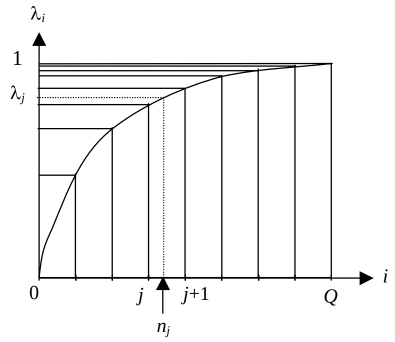

Figure 3: Network loading scheme

Then problem $P_{op}\left(\Phi^{a_i}, \underline{N}^0, \Phi^{d_i}\right)$ is solved where the optimization criterion is replaced by the linear one :

$$\sum_{u \in U} l_u \varphi_u + \sum_{l \in L} \sum_{u \in \omega^-(l)} \sum_{\substack{v \in \omega^-(l) \\ v \neq u}} c_{uv} \left(\varphi_u^{i-1} \varphi_v + \varphi_u \varphi_v^{i-1}\right) \tag{41}$$

where $\varphi_u^{i-1}, u \in \omega^-(l)$ et $\varphi_v^{i-1}, v \in \omega^-(l), v \neq u, l \in L$ are provided by the solution of the same problem at the previous iteration. Now the optimization problem is a linear min cost flow problem in a capacitated network and many efficient solution are available. To start the process, a first feasible solution $\{(\varphi_u^0, \varphi_v^0), u \in \omega^-(l), v \in \omega^-(l), l \in L\}$ is needed. It can be obtained by neglecting the interactions between flows at low flow loadings ($0 < \lambda_0 \leq 1 - \left(\frac{1-n_i}{1+n_i}\right)^p$). Then only the linear term of the original optimization criterion may be considered

The proposed solution process for this flows in network non convex optimization problem being an heuristic one, to get more confidence on the results, the whole process (loading from zero to capacity, solution of the successive linearized optimization problems) is run many times until no improvement is found with new full solutions.

5. APPLICATION

The proposed approach has been applied to Portland International airport using LPSOLVE to solve the linearized versions of the semi loaded flow optimization problems. The following values have been adopted for the loading parameters : Q = 20 and p = 5. Figure 4 shows the layout of the Portland International Airport. Figures 5 and 6 present instances of the dispersion of the final results obtained after successive solving sequences of the above problems (10 iterations each time). Finally in figure 7 the resulting practical capacity envelope is displayed.

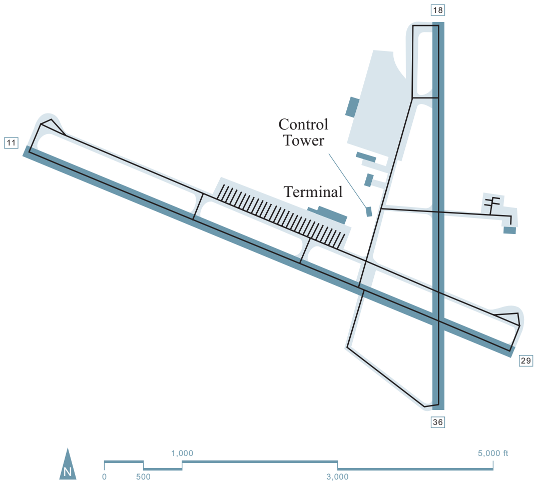

Figure 4: Ground traffic system at Portland International Airport

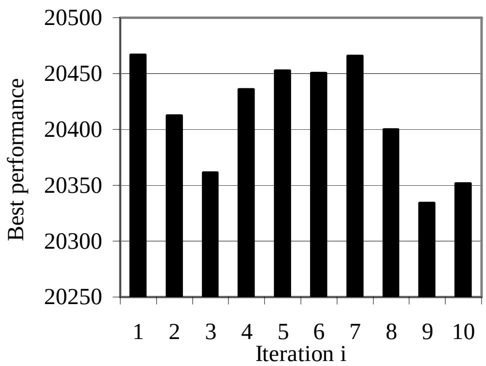

Figure 5: Distribution of performance at full loading ($\Phi^a=\Phi^d=18$)

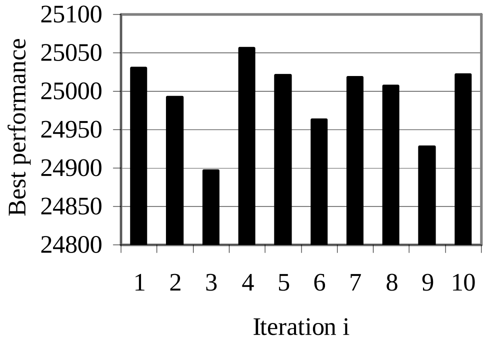

Figure 6: Distribution of performance at full loading ($\Phi^a=\Phi^d=15$)

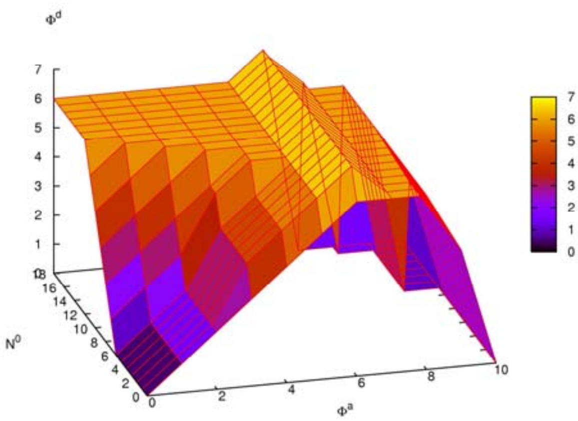

Figure 7: Example of 3D representation of the Portland International Airport practical capacity

6. CONCLUSIONS AND FURTHER WORK

In this communication, a new approach based on the solution of successive optimization problems has been proposed for the estimation of the practical airside capacity of an airport. The level of detail chosen for the representation of aircraft ground traffic movement at airports, through flows in a traffic network, allows the utilisation of the results of the capacity study to tackle many ground operational and planning problems such as the definition of an aircraft circulation plan, the redesign of pre existent taxiways and apron areas and the assignment of parking areas to different airlines.

For given mean in-bound and out-bound flows and a current ground traffic situation, a minimal cost flow problem has been formulated, but, since the interaction of aircraft flows at ground intersections is taken explicitly into consideration, this leads to a non convex optimization problem. A heuristic approach, based on a progressive loading of the network, has been developed to get a solution for the minimal cost flow problem. The proposed heuristic has been assessed numerically, showing acceptable performances and has been applied to the case of Portland International airport for which some numerical results have been displayed. It appears that this approach to solve a non convex optimization problem of flows in a network, could be of interest for other fields of application.

REFERENCES

Andreatta, G., Brunetta, L., Odoni, A.R., Righi, L., Stamatopoulos, M.A. and Zografos, G (1998) *A set of approximate and compatible models for airport strategic planning on airside and on landside*, 2nd USA/Europe ATM R&D Seminar, Orlando, 1-4 December 1998

Andreatta, G. and Romanin-Jacur, G.(1987) *Aircraft flow management under congestion*, Transportation Science, No 21.

Bianco L. et Bielli M. (1992) *Air Traffic Management : Optimization models and algorithms*, Journal of Advanced Transportation Systems, Vol. 26, No. 2.

Carr, F (2000) *Stochastic modelling and control of airport surface traffic*, MSc Thesis Electrical Engineering and Computer Science, MIT, Cambridge, MA.

Stoica D. (2004) *Analyse, représentation et optimisation de la circulation des avions sur une plate-forme aéroportuaire*, Thèse doctorale, INPT, Toulouse, France

Davis Lee, Caroline Nelson and Gerald Shapiro (1998) *The aviation system analysis capability airport capacity and delay models*, NASA/CR-1998-207659, April 1998

FAA(1998) *Traffic management system, chapter 17: traffic management national, centre and terminal* .

Eugene, P. Gilbo (1993) *Airport capacity : representation, estimation, optimisation*, IEEE Transactions on Control Systems Technology, Vol. 1(3).

Lorenzo Brunetta, Luca Righi et Giovanni Andreatta (1999) *An operations research model for the evaluation of an airport terminal : SLAM (simple landside aggregate model)*, Journal of Air Transport Management, No. 5.

Dennis F.X. Mathaisel, Husni Idris (1998) *Aircraft ground movement simulation*. Operations research in the airline industry, ed. G. Yu, Kluwer.

Newell G. F.(1979) *Airport capacity and delays*, Transportation Science, Vol. 13, No. 3.

Polak, F.R.(1996) *Airport modelling : capacity analysis of Schiphol airport in 2015*, 1st USA/Europe ATM R&D Seminar.

STBA (1975) *Capacité du système aéroportuaire* – Manuel de capacité – pistes.