VERIFICATION OF LWR TRAFFIC FLOW THEORY WITH EMPIRICAL DATA

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ABSTRACT
It is common knowledge that traffic dynamic conditions caused by the imbalance between traffic demands and supply increase the uncertainty of traffic services and generates unreliable transportation systems. In this sense, it is very important for traffic control agencies to correctly model the traffic flow in order to better forecast of traffic condition. This work seeks to contribute for understanding and prediction of current and future roadway conditions through the verification of the changes in traffic flow along space and time. Basically, the traditional kinematic wave theory proposed by Lighthill and Whitham (LWR theory) is interpreted and validated with recent empirical data of volume, velocity and occupancy. The data were collected from vehicles detectors strategically located around a bottleneck on a two lane elevated expressway in Japan. Besides, the theory is extended to explain newer empirical evidences owing the improvement of the representation of dynamic driver's behavior.

RESUMO
É de comum conhecimento que as condiçõs dinamicas do tráfego causadas pelo desequilíbrio entre oferta e demanda aumentam as incertezas dos servicos de tráfego e gera sistemas de transporte não confiáveis. Neste sentido, é importante para as agências de controle de trânsito modelar corretamente o fluxo de tráfego no intuito de melhor prever o comportamento do motorista, usando para isto as variáveis fundamentais do fluxo de tráfego. Portanto, este trabalho visa contribuir para o entendimento e previsão das condiçõs atuais e futuras das vias através da verificação das mudanças no comportamento do tráfego ao longo do tempo e do espaço. Basicamente, a tradicional teoria das *kinematic waves* é interpretada e validada para dados empíricos recentes de volume, velocidade e densidade. Os dados foram coletados a partir de detectores de veículos estrategicamente localizados próximos a *bottlenecks* numa via de pista dupla elevada Japonesa. Além disso, a teoria é ampliada para explicar novas evidências empíricas visando o aprimoramento da representação do comportamento dinâmico do motorista.

1. INTRODUCTION

The growth of urban automobile traffic demand has led to serious and worsening traffic congestion problems in most cities around the world. Since the increase in travel demand is often greater than the improvement in road capacity, the situation will continue to deteriorate unless the efficiency of existing road networks is improved (Holland and Woods, 1997).

On the other hand, traffic flow modeling has emerged as important evaluation tools for better representing the interaction between drivers, vehicles and transportation infrastructure in order to increase the road capacities. And as inherent subject, the traffic flow theory through its fundamental variables has received considerable research and practical interests as well.

The most well-known and commonly used theory on the subject of traffic flow modeling was developed more than fifty years ago by Lighthill and Whitham (1955). They introduced the concept of traffic flow dynamics based on the hydrodynamic theory. The theory is usually called as LWR theory (Lighthill-Whitham, 1955 and Richards, 1956). Since then, the description of traffic flow based on their theory has been a lively subject of researches. However, due to the complexity of traffic flow system, in some situations, these descriptions have not providing the desired results. Analysis and tests by simulation models of macro- and micro-properties have pointed out that the results are not consistent with the commonly accepted theoretical flow-concentration/density-speed relationships considered in these descriptions.

As a result, over the years many measurements have been made and various theories based on LWR theory have appeared in the literature. But still there is not a complete agreement among those theories. Examples of the lack of agreement in the development of traffic flow theory can be proved by the existence of many shapes to describe the flow-density relationship. As examples, the classical parabolic (inverted U) relationship between flow rate-concentration presented by Greenshields in 1935 (*apud* Banks, 1989) differs from the 'reversed lambda' model proposed by Koshi *et al*. in 1983 (*apud* Banks, 1989). Hall and Gunter (1986) also give contribution in this area by showing an inverted V shape to the relationship. Besides, assumptions about the discontinuity of the curve representing the relationship are attributed to Edie, 1961. These assumptions suggest that curves separated by regimes, congested and non-congested, might be needed for the flow-concentration/density relationship.

An alternative way of looking at these disagreements does not focusing on the nature of the relationship but on the mismatch between the classical theory and the recent empirical observations. According to Gilchrist and Hall (1989), improved data collections methods have often led to serious questioning of traditional theory. Moreover, another point to consider is the area and location of data observation. As pointed out by May (1990), the nature of the data acquired from a freeway depends on where the measurements are taken, especially with respect to bottlenecks and queues. By Banks (1989), data might have been misinterpreted in the past due to the lack of sensitivity to the effects of location on freeway flow phenomena. Findings from Hall and Gunter (1986) confirm that the relationship varies according to both lane and location.

Consequently, the challenge of traffic flow researches is to reach a compromise or improve the existing theory in order to make it able to characterize the traffic flow dynamics accurately. In this respect, two are the objectives of this study. The first objective is to validate the kinematic wave theory developed by Lighthill and Whitham in 1955 and Richards, 1956 with recent empirical data in order to identify the points of agreements and disagreements between theory and reality. The second objective is the extension of the theory to explain newer empirical evidences of traffic flow. For the study, the data used were collected by vehicle detectors strategically located before, on and after a bottleneck on a two lane elevated expressway.

2. KINEMATIC WAVE THEORY

The concept of 'Kinematic Waves' was introduced originally by Lighthill and Whitham in 1955. In their paper 'On Kinematic Waves', they described a theory of one-dimensional wave motion which could be applied to certain types of fluid motion or to highway traffic flow. The key postulate of LWR theory is that there is a functional relation between flow, q (quantity passing at a given point in unit time), concentration or density, k (quantity per unit distance) and position x. This relation might vary with position x, but not with time. As dynamic waves, the kinematic wave's property follows from the equation of continuity, that is, the conservation law, supplemented by the definition of flow rate, and the equilibrium relationship between speed and traffic density.

By considering the flow-concentration/density relationship, the basic conclusion of kinematic wave theory states that information obtained from low and high values of concentration should be combined into a single curve. This curve is a graph of the two fundamental quantities, flow (q) against concentration (k). An example of such curve can be illustrated as in Figure 1. As the concentration k tends to zero, the flow q must also become zero. In the limiting case of high concentration $k=k_j$ (j for jam) the vehicles traveling in a given direction are packed tight on the part of the road where they are permitted to be, and the flow q is then again zero. For some value of the concentration between these two extremes, the flow q must have a maximum value, q_m, which is usually denoted as a capacity of the road.

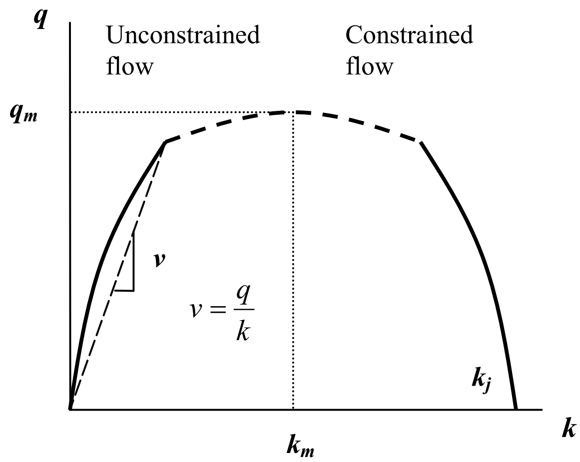


Figure 1: Flow-concentration curve to a stretch of a road (modified by Lighthill and Whitham, 1955)

When successive kinematic waves combine, another wave called 'kinematic shock wave' arises. A shock wave is the boundary between two density states. Among other reasons, two density states appear due to a presence of a bottleneck, that is, where the capacity of the road varies or where the capacity of a uniform subsection of a road is lower than the capacity of the main section.

3. INTERPRETATION OF LWR THEORY

The theory presented in the previous section is interpreted on the basis of a hypothetical case in order to produce a reasonable description of traffic flow characteristics when an increased flow rate reaches a bottleneck. Subsequently, it is assumed that in the hypothetical case, the conditions of traffic volume, capacity and q-k relationship are described as in Figure 2. Sites A and C are located upstream and downstream of the bottleneck respectively, where the road is assumed to have the same capacity. Site B is on the bottleneck. The traffic capacity of each site is defined as Q_c^A, Q_c^B and Q_c^C, respectively. The interpretation of q-k relationship in sites A, B and C when changes occur in the traffic volume is illustrated in Figure 3.

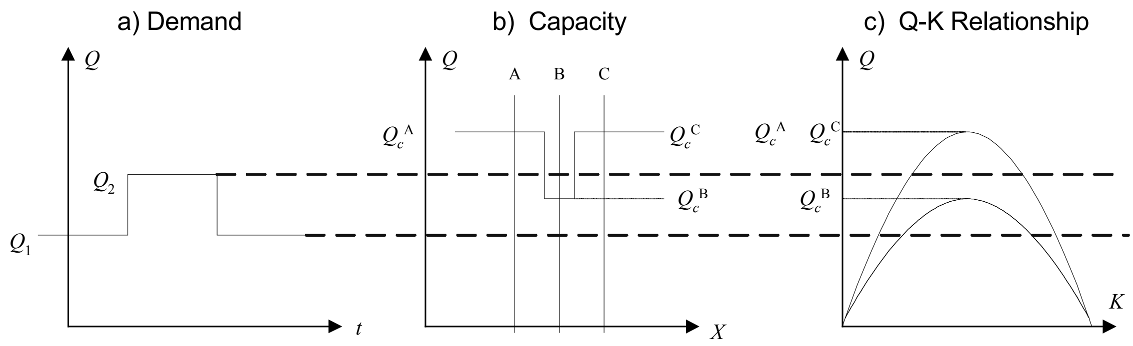


Figure 2: Conditions of traffic volume, capacity and *q*-*k* relationship for a hypothetical situation

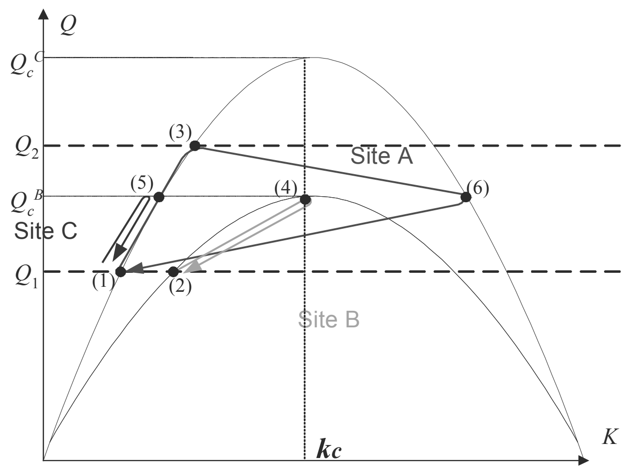


Figure 3: Variation in the *q*-*k* relationship for the sites A, B and C

We will then explore how the traffic condition of each point changes when incoming demand from upstream changes. We assume that initially incoming traffic demand, Q_1 is less than the bottleneck capacity (Q_c^B). After some time goes by, the oncoming flow rate increases to above the capacity of the bottleneck Q_2. The interpretation of *q*-*k* relationship in A, B and C when changes occur in the traffic volume are illustrated in Figure 3. At site A, when incoming demand is Q_1, traffic condition corresponds to the point (1) in Figure 3. Similarly, traffic condition at sites B and C correspond to points (2) and (1) in Figure 3, respectively. Because inflowing demand is less than the capacity of bottleneck, no congestion occurs. Once inflowing demand increases to Q_2, traffic condition on site A changes from (1) to (3). When a wave of Q_2 arrives at the bottleneck, because the bottleneck can not afford Q_2, backward shockwave comes into being. In this situation, at site B, maximum capacity, Q_c^B should be observed. This traffic condition corresponds to point (4) on Figure 3. By contrast, as traffic flowing out from the bottleneck is Q_c^B, traffic condition at site C changes from (1) to (5). When backward shock wave created at site B reaches to site A, the traffic condition of site A suddenly changes from (3) to (6) in Figure 3. The backward shockwave moves upstream until inflowing traffic demand decreases less than the bottleneck capacity. From Figure 3, the transition of traffic flow at each site can be summarized as follows: Traffic condition at site A changes suddenly from (3) to (6) and traffic condition whose flow rate is around Q_c^A can not be observed. It can also be said that the plots representing constrained and non-constrained conditions are located on their own *q-k* curve; regardless of the traffic condition at any other locations. Considering site B, the demand varies from Q_1 to the capacity of the bottleneck, which means from non-constrained flow to near-capacity flow. Constrained flow can not be observed at the bottleneck. After the bottleneck, or at site C, the traffic flow conditions do not present remarkable changes since during all the time the traffic conditions are non-constrained.

Eventually, it is possible to reconfirm three aspects derived from LWR theory:
1) The flow-occupancy/density curve is particular for a stretch of a road, which means that the relationship between *q* and *k* may varies with its location and the relationship in one location is independent from that of others,
2) The maximum traffic volume of the road is determined based on the capacity of the bottleneck and,
3) Non-constrained flow condition can not be observed on and downstream of the bottleneck.

4. VALIDATION OF LWR THEORY WITH EMPIRICAL DATA

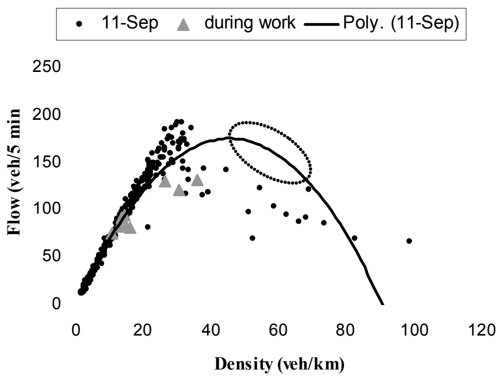
Three aspects obtained at the previous section are validated using empirical data. Occupancy instead of density was used because it is commonly measured by freeway management systems. Density can be measured only along a length. If only point measurements are available, density needs to be estimated from either occupancy or from speed and flow (Hall, 1992). Hall and Persuad (1989) indicate that the nonlinear aspect of the relationship between occupancy and density depends on the covariance between the vehicles' lengths and vehicles' speeds. Due to minor magnitude of nonlinearity and level of generality considered here, density was estimated from occupancy measurement.

To validate the findings of the previous section, we need empirical data around the bottleneck. Especially, observation around an apparent bottleneck such as where number of lanes decreases is preferable. Therefore, we picked up the data observed when one of two lanes was closed because of maintenance work. As duration of the maintenance work was not so long, we could not get enough data to obtain statistical distributions of traffic conditions. Therefore, we also explore data obtained at merging section, where traffic congestion is observable every day, although merging section is it not an apparent bottleneck.

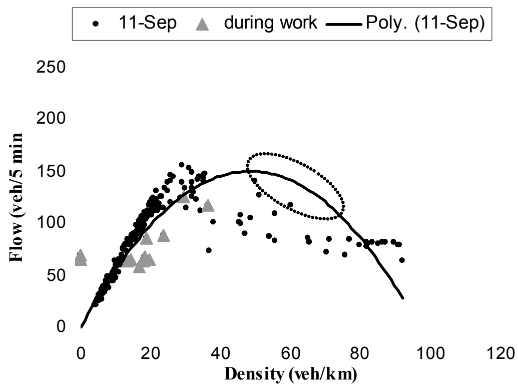
Five-minute interval data of one day collected by vehicle detectors located before, during and after a bottleneck were analyzed. For technical reasons, from now the locations of the detectors before, on and after the bottleneck are named 36.1kp (kilo-post), 35.5kp and 34.9kp respectively. As first case considered, the apparent bottleneck was caused by a maintenance work in one of the lanes. The work was implemented from 10:10 to 11:15 in the morning on a weekday.

Flow-concentration relationship

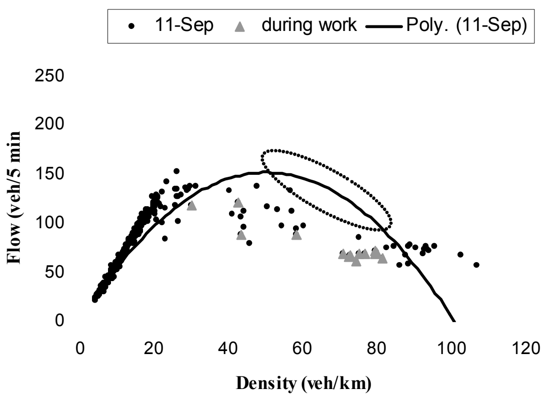
To observe how flow (*q*) and density (*k*) vary over space and time around bottlenecks, scatter of flow versus occupancy plots of the data at 34.9, 35.5 and 36.1 kp locations were displayed in Figure 4. According to LWR theory, the plots might lay 'on' the *q-k* curve of their locations and this curve, which means the relationship between flow and occupancy, is peculiar to that spot and independent of other locations. Figure 4 suggests an agreement with the theory that the *q-k* curve is different with location. Moreover, It shows that the plots are located on their own *q-k* curve in some sense, especially non-congested situation. However, there are also some disagreement that we have some plots below the regression curve connecting non-congested situation and congested situation. The plots do not follow the full line of the curve and an area on the upper right side presents lack of observation. This can be due to interval used to aggregate the data which produces "average" densities between low densities (free flow condition) and high densities (congested condition).



a) 34.9 Kp (after the bottleneck)



b) 35.5 Kp (on the bottleneck)



c) 36.1 Kp (Before the bottleneck)

Figure 4: Flow-occupancy curves of different spots

To explore the shape of *q*(*k*) curve using more data of congested flow, 30 seconds interval data of volume and occupancy, in addition to classification of congested and non-congested regime were used and they were provided by one vehicle detector installed upstream of an on-ramp (merging section). It is illustrated in Figure 5. By the Fig 5, it is possible to recognize a trend for separation in the results. Perhaps this trend can be interpreted as a discontinuity in *q-k* curve since both data, non-constrained and constrained scatter in quite distinct areas. Nevertheless, any assumption within this concern might be confirmed by a complete overview about the full range of traffic movements for one spot.

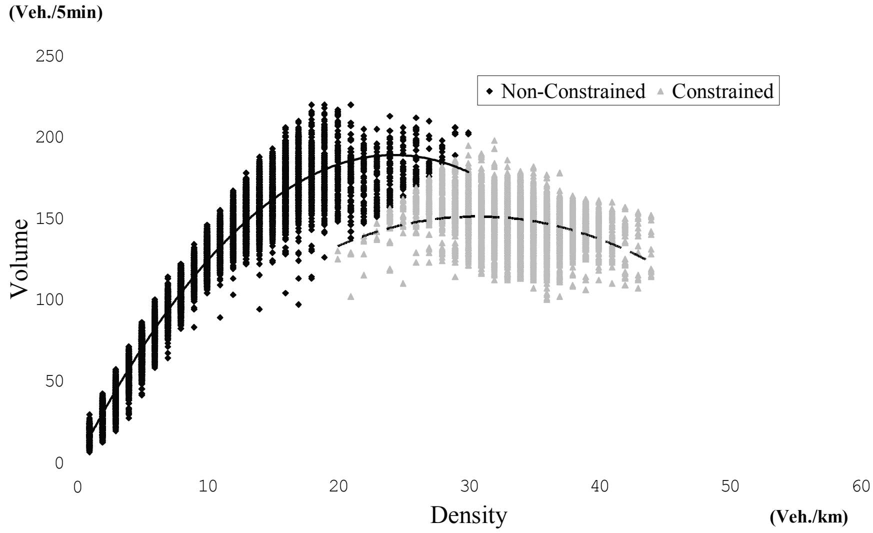


Figure 5: Flow-density curve from on-ramp data

Road Capacity
Considering the capacity of the road, another finding obtained from Figure 4 is that we have little observation of congested flow on the downstream of the bottleneck (Figure 4(a)), whereas we have a bunch of plots around flow of 50(veh/5min) and density of 90 (veh/km) on the upstream of the bottleneck (Figure 4c)). In addition, highest concentrations of plots during the work (represented by triangles in Figure 4) are located around 50(veh/5min) on the curve for the three locations, before, on and after the bottleneck. It indicates that the capacity of the road next to a bottleneck is determined by the capacity of the bottleneck.

Traffic conditions
In another analysis, values of space mean speed occurring simultaneously at different locations (Figure 6) was computed to determine the traffic conditions near to the bottleneck and hence to validate the third findings from LWR theory. At the same time, values of density occurring simultaneously at different locations (Figure 6) were observed to carry through an analysis of the relation between space mean speed and density owing the confirmation of the traffic conditions near to a bottleneck. By May (1990), it is possible to recognize three regions of traffic flow conditions in terms of density: non-congested flow conditions for 0-26 vehicles/lane-km; near-capacity flow conditions for 26-42 vehicles/lane-km and congested flow conditions for value higher than 42 vehicles/lane-km. By the Figure 6, upstream of a bottleneck there is constrained flow; within the bottleneck section near-capacity flow occurs but not congested operations and, downstream of the bottleneck there is neither congested nor capacity operation.

5. CONCLUSION AND DISCUSSION
This study discussed the theory concerning speed-flow-density relationship that has appeared long time ago in order to confirm some ideas based on recent data. The conclusion is that the theory is still in agreement with the recent empirical data. In other words, it is found that the theory presented above is appropriate to describe the traffic flow around bottleneck though some limitations regarding data collection could be identified. Measurements taken from

vehicle detectors are based on observations at one point or at a short section of the road. This method of data collection presents shortcomings associated to a lack of overview about traffic movements. The full range of traffic operations including the piling up of vehicles from the beginning of a bottleneck until it reaches the detector point is unobservable. Consequently, from this study it is proved that location and equipments used to take measurements play an important role on the nature of the data. Therefore, 'spatial' observation rather than 'point' observation is needed for better understanding of traffic flow dynamics.

values of velocity

time	9:35	9:40	9:45	9:50	9:55	10:00	10:05	10:10	10:15	10:20	10:25	10:30	10:35	10:40	10:45	10:50	10:55	11:00	11:05	11:10	11:15	11:20	11:25	11:30	11:35	11:40	11:45	11:50	11:55	12:00
34.9kp	64	66	65	64	14	54	27	45	46	73	78	80	73	76	76	77	76	77	73	47	57	45	18	23	16	36	21	16	59	16
35.5kp	61	59	55	56	36	26	41	60	63	44	43	45	43	45	44	42	45	58	49	42	57	58	20	12	15	23	29	11	49	36
36.1kp	123	113	110	111	120	52	108	49	22	22	23	22	19	21	22	21	22	20	36	68	39	114	64	19	39	16	51	16	43	87

one lane closed by maintenance work

legend: 60km/h- | 40-60km/h | 20-40km/h | 0-20km/h

values of density

time	9:35	9:40	9:45	9:50	9:55	10:00	10:05	10:10	10:15	10:20	10:25	10:30	10:35	10:40	10:45	10:50	10:55	11:00	11:05	11:10	11:15	11:20	11:25	11:30	11:35	11:40	11:45	11:50	11:55	12:00
34.9kp	31	32	31	31	74	32	55	31	21	10	13	10	12	11	12	13	12	11	15	34	32	38	63	51	53	39	59	68	27	66
35.5kp	34	33	36	33	36	55	37	21	15	26	39	35	37	35	37	37	33	16	26	38	33	32	63	81	66	49	48	79	30	44
36.1kp	26	27	30	26	21	44	23	62	96	73	71	77	81	83	73	92	87	76	77	48	30	27	41	96	58	89	53	86	55	23

one lane closed by maintenance work

legend: 0-26veh/km | 26-42veh/km | 42veh/km -

Figure 6: Values of velocity and density according to time and locations

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